IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X.
PP 18-21
www.iosrjournals.org

Solving Multi-Objective Fuzzy Linear Optimization Problem Using Fuzzy Programming Technique

Beena T Balan
Dept of Science and Humanities SNGCE, Kadayiruppu ,kolencheri Ernakulam &
Research Scholar Karunya university, Coimbatore

Abstract : *Fuzzy multi objective linear programming problem has its application in a variety of research and development field. It is the application of fuzzy set theory in linear decision making problem. In this paper, solution procedure of multi objective fuzzy linear programming with triangular membership function is presented. With the help of numerical example the method is illustrated.*

Keywords : *Fuzzy Linear Programming, Fuzzy number , Optimal solution, pareto optimality, Triangular Fuzzy number .*

I. Introduction

The real world problems are too complex, and the complexity involves the degree of uncertainty in the form of ambiguity, vagueness, chance or incomplete knowledge. Most of the parameters of the problem are defined by means of language statements. Therefore it will give better result if we consider the Decision makers knowledge as fuzzy data. Fuzzy modelling provides a mathematical way to represent vagueness and fuzziness in humanistic system.

The idea of fuzzy set was first proposed by Zadeh [10] as a mean of handling uncertainty that is due to imprecise rather than to randomness. After that Bellman and Zadeh [1] proposed that a fuzzy decision might be defined as the fuzzy set, defined by the intersection of fuzzy objective and constraint goals. Then Zimmermann [5] introduced fuzzy linear programming problem in fuzzy environment. Gasimov and Yenilmez [11] considered single objective mathematical programming with all fuzzy parameters. In their paper coefficients of constraints were taken as fuzzy numbers. They solved it by fuzzy decisive set method and modified sub-gradient method. Zimmermann proposed a fuzzy multi criteria decision making set, defined as the intersection of all fuzzy goals and constraints. Tanaka and Asai [4] considered the fuzzy Linear programming problem with fuzzy decision variables and crisp decision parameters. Chanas [2] proposed a fuzzy programming in Multi Objective Linear Programming problem and it was solved by possibility distribution.

In this paper, solution procedure of multi objective fuzzy linear programming and its application is presented. Here the coefficients of objective and constraint functions are represented as Triangular fuzzy number. The problem is then converted to crisp linear programming problem based on the concept of fuzzy decision and the fuzzy model under fuzzy environments proposed by Bellman and Zadeh (1970). It is solved using Pareto's Optimality techniques.

II. Notation and basic definition

Definition: Let X be a classical set of objects which should be evaluated with regards to a fuzzy statement. Then the set of ordered pairs $\tilde{A} = \{(x, \mu_A\ (x)) | x \in X\}$
where μ_A :X→ $[0,1]$ is a fuzzy set in X. The evaluation function $\mu_A\ (x)$ is called the membership function or the grade of membership of x in $\tilde{A}$

Definition: A Fuzzy set $\tilde{A}$ is called normalised fuzzy set if $Sup_{x \in X}\ \mu_A\ (x)$ =1

Let $\tilde{A}$ be a fuzzy set in X and $\alpha \in [0,1]$ a real number. Then the classical set $A_\alpha = \{x \in X | \mu_A\ (x) \geq \alpha\}$ is called α-level set or α-cut of$\tilde{A}$, A_α = {x∈X $|\ \mu_A$ (x)> α} is called strong α-cut of $\tilde{A}$

Definition: A fuzzy set $\tilde{A}$ in a convex set X is called convex if
$\mu_A\ (\lambda x_1$+(1- λ) $x_2) \geq$ Min $(\mu_A(x_1), \mu_A(x_2))$, $x_1, x_2 \in X$, $\lambda \in [0,1]$

Definition: A convex normalised fuzzy set $\tilde{A}$ =$\{(x, \mu_A\ (x)) | x \in R\}$ on the real line R such that i) there exist exactly one $x_0 \in R$ with the membership degree $\mu_A(x_0)$ =1 and ii) $\mu_A\ (x)$ is piecewise continuous in R, is called a fuzzy number.

A convex normalised fuzzy set $\tilde{A} = \{(x, \mu_A\ (x)) | x \in R\}$ on the real line R is called a fuzzy interval if i) there exists more than one real number x with a membership degree $\mu_A\ (x)$ =1. ii) $\mu_A\ (x)$ is piecewise continuous in R

Triangular Fuzzy Number: A fuzzy number $\tilde{A}$ is a triangular fuzzy number denoted by (c, l, r) where c, l, r are real numbers and c is the peak value of $\tilde{A}$ and membership function is given by

$\mu_A\ (x)\ = \frac{x-c+l}{l}$ for $c-l \le x \le c$

$= \frac{c+r-x}{r}$ for $c \le x \le c+r$

$= 0$ elsewhere

Consider two fuzzy number $\tilde{A}$ and $\tilde{B}$ where $\tilde{A} = (c_1, l_1, r_1)$ and

$\tilde{B} = (c_2, l_2, r_2)$

$\tilde{A} + \tilde{B} = (c_1+c_2, l_1+l_2, r_1+r_2)$

$k\tilde{A} = kc_1, kl_1, kr_1$ for $k \ge 0$

Definition: The partial ordering $\le$ is defined by $\tilde{A} \le \tilde{B}$ iff

$c_1 \le c_2$, $c_1-l_1 \le c_2-l_2$, $c_1 + r_1 \le c_2+r_2$

Theorem: (Pareto Optimality) For a problem with k objective functions, The point x* ∈X is a Pareto Optimal solution if there does not exist x∈X such that if $Z_i\ (x) \ge Z_i\ (x^*)$ for all i and

$Z_j\ (x) > Z_j\ (x^*)$ for at least one j.

III. Linear Programming Problems with Fuzzy Parameters

The Linear Programming problem with fuzzy parameters are given as

Maximize $\tilde{Z} = \sum_{j=1}^{n} \tilde{c}_j\ x_j$

Such that

$\sum_{j=1}^{n} \tilde{a}_{ij}\ x_j \le \tilde{b}_i$ For i=1, 2,3, ... m

$x_j \ge 0$, j =1,2,3....n.

Where $\tilde{c}_j, \tilde{a}_{ij}, \tilde{b}_i$ are fuzzy numbers.

Considering the fuzzy parameters as triangular fuzzy numbers

Maximizing $\tilde{Z} = \sum_{j=1}^{n} (cc, cl, cr)_j\ x_j$

Such that

$\sum_{j=1}^{n} (ac, al, ar)_{ij}\ x_j \le (bc, bl, br)_i$ i =1, 2,3....m

$x_j \ge 0$ j=1, 2,3... n

where $(cc, cl, cr)_j$ is the j^th fuzzy coefficient in the objective function. $(ac, al, ar)_{ij}$ is the fuzzy coefficient of j^th variable in the i^th constraints, $(bc, bl, br)_i$ is the i^th fuzzy resource .These problem can be solved by converting into equivalent crisp multi –objective linear problem

Maximize (Z_1, Z_2, Z_3)

Where $Z_1 = \sum_{j=1}^{n} cc_j\ x_j$ $Z_2 = \sum_{j=1}^{n} cl_j\ x_j$ $Z_3 = \sum_{j=1}^{n} cr_j\ x_j$

Such that $\sum_{j=1}^{n} ac_{ij}\ x_j \le bc_i$, i= 1, 2,3...m

$\sum_{j=1}^{n} (ac_{ij} - al_{ij}) x_j \le bc_i - bl_i$, i= 1, 2, 3...m

$\sum_{j=1}^{n} (ac_{ij} + ar_{ij})\ x_j \le bc_i + br_i$, i= 1, 2, 3...m

$x_j \ge 0$ j=1, 2, 3... n

IV. Multi –objective fuzzy linear programming Problem

Maximize $\tilde{Z}^1, \tilde{Z}^2, \tilde{Z}^3, \dots\dots \tilde{Z}^k$

Where $\tilde{Z}^k = \sum_{j=1}^{n} \tilde{c}_j^k x_j$, k=1,2,3,...k

Such that

$\sum_{j=1}^{n} \tilde{a}_{ij}\ x_j \le \tilde{b}_i$ For i=1,2,3, ...m

$x_j \ge 0$, j =1,2,3....n.

Where $\tilde{c}_j^k$, $\tilde{a}_{ij}, \tilde{b}_i$ are fuzzy numbers and x_j are fuzzy variables.

Considering the ob[jective function coefficient, technological coefficient and resource coefficient as triangular fuzzy numbers.

The above Problem can be represented as

Maximize $\tilde{Z}^1, \tilde{Z}^2, \tilde{Z}^3, \ldots\ldots \tilde{Z}^k$
where $\tilde{Z}^k = \sum_{j=1}^{n} (cc, cl, cr)_j^k x_j$, k=1,2,3,...k
Such that
$\sum_{j=1}^{n} (ac, al, ar)_{ij} \; x_j \leq (bc, bl, br)_i$ for i=1,2,3, ...m
$x_j \geq 0$, j =1,2,3....n.
$(cc, cl, cr)_j^k$ is the j[th] fuzzy coefficient in the k[th] objective function $(ac, al, ar)_{ij}$ is the fuzzy coefficient of j[th] variable in the i[th] constraint, $(bc, bl, br)_i$ is the i[th] fuzzy resource.

V. Solution for Multi objective fuzzy linear Programming Problem

Here there are k fuzzy objective function $\tilde{Z}^1, \tilde{Z}^2, \tilde{Z}^3, \ldots\ldots \tilde{Z}^k$ and m fuzzy constraints $\tilde{G}^1, \tilde{G}^2, \tilde{G}^3, \ldots\ldots \tilde{G}^m$ where $\tilde{G}^i = \sum_{j=1}^{n} \tilde{a}_{ij} \; x_j \leq \tilde{b}_i$ for i=1,2,3, ...m. The resulting fuzzy decision is given as $\tilde{Z}^1 \cap \tilde{Z}^2 \cap \tilde{Z}^3 \cap \ldots \cap \tilde{Z}^k \cap \tilde{G}^1 \cap \tilde{G}^2 \cap \tilde{G}^3, \ldots \cap \tilde{G}^m$. In terms of corresponding membership values for the fuzzy objectives and fuzzy constraints the decision is
$\mu_{\tilde{D}}(x) = \min_{k,i}(\mu_{\tilde{Z}^K}(x) \, , \mu_{\tilde{G}^i}(x))$
An Optimal solution X^* is one at which the membership function of the resultant decision $\tilde{D}$ is maximum.
i.e. $\mu_{\tilde{D}}(X*) = \max(\min_{k,i}(\mu_{\tilde{Z}^K}(x) \, , \mu_{\tilde{G}^i}(x)))$

Here the coefficient of objective function is represented by triangular fuzzy numbers. Therefore each objective function gives rise to 3 crisp objective functions. Hence the converted problem will involve 3k objective function to be optimised.
Maximize $(Z_1^1, Z_2^1, Z_3^1, Z_1^2, Z_2^2, Z_3^2 \ldots\ldots\ldots Z_2^k, Z_3^k)$
Where $Z_1^k = \sum_{j=1}^{n} (cc)_j^k x_j$;
$Z_2^k = \sum_{j=1}^{n} (cl)_j^k x_j$;
$Z_3^k = \sum_{j=1}^{n} (cr)_j^k x_j$;
Such that $\sum_{j=1}^{n} ac_{ij} \; x_j \leq bc_i$, i= 1,2,3...m
$\sum_{j=1}^{n} (ac_{ij} - al_{ij}) x_j \leq bc_i - bl_i$, i= 1,2,3...m
$\sum_{j=1}^{n} (ac_{ij} + ar_{ij}) x_j \leq bc_i + br_i$, i= 1,2,3...m
$x_j \geq 0$ j=1, 2, 3... n
The weighted objective function of the above problem using pareto method can be represented as
Max $w_{11} Z_1^1 + w_{12} Z_2^1 + w_{13} Z_3^1 + w_{21} Z_1^2 + w_{22} Z_2^2 + \ldots\ldots\ldots + w_{k3} Z_3^k$
Where $Z_1^k = \sum_{j=1}^{n} (cc)_j^k x_j$;
$Z_2^k = \sum_{j=1}^{n} (cl)_j^k x_j$;
$Z_3^k = \sum_{j=1}^{n} (cr)_j^k x_j$;
Such that $\sum_{j=1}^{n} ac_{ij} \; x_j \leq bc_i$, i= 1,2,3...m
$\sum_{j=1}^{n} (ac_{ij} - al_{ij}) x_j \leq bc_i - bl_i$, i= 1,2,3...m
$\sum_{j=1}^{n} (ac_{ij} + ar_{ij}) x_j \leq bc_i + br_i$, i= 1,2,3...m
$x_j \geq 0$ j=1,2,3... n
Solution can be obtained by solving the problem with different weights.

VI. Numerical example

Consider the Multi objective fuzzy linear Problem
Maximize $\tilde{Z}^1$ = (7,10,14) x_1+ $(20,25,35)_2 x_2$
$\tilde{Z}^2$ = (10,14,25)x_1+ (25,35,40)x_2

Subject to the constraints
(3,2,1) x_1+(6,4,1) x_2 $\leq$ (13,5,2)
(4,1,2) x_1+(6,5,4) $x_2 \leq$ (7,4,2)
Where the membership function of the $\tilde{c}_1$, $\tilde{c}_{2,}$, $\tilde{c}_3$, $\tilde{c}_4$ are

$$\mu_{\tilde{c}1}(x) = \begin{cases} \frac{x-7}{3} & for\ 7 < x \leq 10 \\ \frac{14-x}{4} & for\ 10 < x \leq 14 \\ 0 & Elsewhere \end{cases}$$

$$\mu_{\tilde{c}2}(x) = \begin{cases} \frac{x-20}{5} & for\ 20 < x \leq 25 \\ \frac{35-x}{10} & for\ 25 < x \leq 35 \\ 0 & Elsewhere \end{cases}$$

$$\mu_{\tilde{c}3}(x) = \begin{cases} \frac{x-10}{4} & for\ 10 < x \leq 14 \\ \frac{25-x}{9} & for\ 14 < x \leq 25 \\ 0 & Elsewhere \end{cases}$$

$$\mu_{\tilde{c}4}(x) = \begin{cases} \frac{x-25}{5} & for\ 25 < x \leq 35 \\ \frac{40-x}{5} & for\ 35 < x \leq 40 \\ 0 & Elsewhere \end{cases}$$

It is equivalent to solving the Multi objective linear programming problem

Maximize $\begin{pmatrix} 7x1 + 20x2, & 10x1 + 25x2 \\ 14x1 + 35x2 & 25x1 + 40x2 \end{pmatrix}$

Subject to the constraints

$3x_1+6x_2 \leq 13$
$x_1+2x_2 \leq 8$
$4x_1+7x_2 \leq 15$
$4x_1+6x_2 \leq 7$
$3x_1+x_2 \leq 3$
$x_1+10x_2 \leq 9$
$x_1, x_2 \geq 0$

This is same as solving
Maximize w1 $(7x1 + 20x2)$+w2$(10x1 + 25x2)$+w3$(14x1 + 35x2)$ +w4$(25x1 + 40x2)$
Subject to the above constraints
Such that $\sum w_i = 2$

Standard Optimization technique is used to solve the problem by using different weights in such that their sum is 2and solving the resultant single objective Linear programming problem. For example by taking w1 =0 and w4 =0 and w2 =w3=1 we get Multi objective linear programming problem.
Solving we get(x_1, x_2) =(0,0.9).

VII. Conclusion

In this paper solution procedure of multi objective fuzzy linear programming and its application were presented. Here the coefficient of objective and constraint functions was represented as Triangular fuzzy number. The problem is then converted to crisp linear programming problem based on the concept of fuzzy decision and the fuzzy model under fuzzy environments proposed by Bellman and Zadeh (1970).

References

.
[1] R.E. Bellman, and L.A. Zadeh, Decision making in a fuzzy environment, *Management Science 17*,1970 141-164.
[2] D. Chanas, Fuzzy programming in multi objective linear Programming-a parametric approach, *Fuzzy Set and system ,29* ,1 989 303-313.
[3] H. Ishibuchi H. Tanaka Multi-objective programming in optimization of the interval objective function, *European Journal of Operational Research , 48*, 1990, 219-225.
[4] H., Tanaka ,and K., Asai, Fuzzy linear programming problems with fuzzy numbers, *Fuzzy Sets and Systems*, *13*, 1984, 1-10.
[5] H.J., Zimmermann, Fuzzy programming and linear programming with several objective functions, *Fuzzy sets and System* , 1978, 45- 55.
[6] S.H.Nasseri, A New Method for Solving Fuzzy Linear Programming by Solving Linear Programming. *Applied Mathematics sci., 2(50),* 2008, 2473-2480.
[7] Makani Das and ,Hemanta K.Baurah, Solution of A linear Programming Problem with Fuzzy Data.*The Journal of Fuzzy Mathematics ,12(4),* 2004, 793-811.
[8] G.J. Klir. and T.A Folger. *Fuzzy sets,* Uncertainty and information. (Prentice hall, New York, 1988).
[9] H. Rommelfanger,Fuzzy linear Programming and applications. European *Journal of Operational Research, , 92*, 1996 .512-527.
[10] L.A., Zadeh, Fuzzy sets, *Information and Control*, 8, 1965 69-78.
[11] R.N. Gasimov and K.Yenilmez, Solving fuzzy linear programming with linear membership functions. *Turk J. Math.26* 2002 375-396.